ABSTRACT BOOKLET

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Università degli Studi di Catania

Third Workshop on

Variational Inequalities, Nash Equilibrium Problems and Applications

Catania, 6-7 October 2016

Variational Inequalities, Nash Equilibrium Problems and Applications

The third edition of the workshop VINEPA aims at presenting the state-of-the-art and current research on variational inequalities, Nash equilibrium problems and applications.

Recently, there has been a sharp increase in interest in variational inequalities, that have become one of the most challenging and dynamic topics of mathematics, and represent excellent tools in the study of real world problems. In fact, they cover a large variety of applications of extreme importance related to computer science, mathematical physics, engineering, statistics, economics, financial networks and generalized complementarity problems.

In addition, Nash equilibrium problems, naturally associated with variational inequalities, experienced a surprising development. This leads to finding effective solutions to until now unsolved problems in numerous real life situations, such as oligopoly models, environmental problems, network problems, multiuser communication systems and infrastructure problems.

Variational methods and game theory are also used in computer vision, machine learning and pattern recognition, in order to obtain intelligent systems able to reconstruct the geometric structure of a scene starting from images; to infer the motion of objects in videos; to make the semantic partitioning of pixels in an image, and make it possible to learn and infer visual data using Bayesian models based on graphs.

The aim of the workshop is to bring together scholars working on both theoretical and computational issues, present results having the potential of solving concrete problems, and thus try to fill the gap between theory and practice.

This booklet contains the abstracts of the communications presented during VINEPA 2016.

Catania, September 2016

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INVITED COMMUNICATIONS

Evolution Problems Involving Nonconvex Moving Sets

Adly S.

Abstract: In this talk, we analyze and discuss the well-posedness of the so-called sweeping process, introduced by J.J. Moreau in the early 70's with motivation in plasticity theory. The first variant is concerned with the perturbation of the normal cone to the moving convex (or prox-regular) subset $C(t)$, supposed to have a bounded variation, by a Lipschitz mapping. Under some assumptions on the data, we show that the perturbed differential measure inclusion has one and only one right continuous solution with bounded variation. The existence of a solution with bounded variation is achieved thanks to the Moreau's catching-up algorithm adapted to this kind of problem. Various properties and estimates of jumps of the solution are also provided. We give sufficient conditions to ensure the uniform prox-regularity when the moving set is described by inequality constraints. As an application, we consider a nonlinear differential complementarity system which is a combination of an ordinary differential equation with a nonlinear complementarily condition. Such problems appear in many areas such as nonsmooth mechanics, nonregular electrical circuits and control systems. The second variant concerns a first order sweeping process with velocity in the moving set $C(t)$. This class of problems subsumes as a particular case, the evolution variational inequalities (widely used in applied mathematics and unilateral mechanics).

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Variational Methods and Geometric Optimization for Computer Vision

Cremers D.

Abstract: Variational methods are among the most classical and established methods to solve a multitude of problems arising in computer vision and image processing. Over the last years, they have evolved substantially, giving rise to some of the most powerful methods for optic flow estimation, image segmentation and 3D reconstruction, both in terms of accuracy and in terms of computational speed. In this tutorial, I will introduce the basic concepts of variational methods. I will then focus on problems of geometric optimization including image segmentation and 3d reconstruction. I will show how the regularization terms can be adapted to incorporate statistically learned knowledge about our world. Subsequently, I will discuss techniques of convex relaxation and functional lifting which allow to computing globally optimal or near-optimal solutions to respective energy minimization problems. Experimental results demonstrate that these spatially continuous approaches provide numerous advantages over spatially discrete (graph cut) formulations, in particular they are easily parallelized (lower runtime) and they do not suffer from metrication errors (better accuracy).

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The Regularity of 2-Dimensional Area-Minimizing Integral Currents

De Lellis C.

Abstract: Building upon the Almgren's big regularity paper, Chang proved in the eighties that the singularities of area-minimizing integral 2-dimensional currents are isolated. His proof relies on a suitable improvement of Almgren's center manifold and its construction is only sketched. In recent joint works with Emanuele Spadaro and Luca Spolaor we give a complete proof of the existence of the center manifold needed by Chang and extend his theorem to two classes of currents which are "almost area minimizing", namely spherical cross sections of area-minimizing 3-dimensional cones and semicalibrated currents.

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A Game-Theoretic Approach to Computation Offloading in Mobile Cloud Computing

Facchinei F.

Abstract: We consider a three-tier architecture for mobile and pervasive computing scenarios, consisting of a local tier of mobile nodes, a middle tier (cloudlets) of nearby computing nodes, typically located at the mobile nodes access points but characterized by a limited amount of resources, and a remote tier of distant cloud servers, which have practically infinite resources. This architecture has been proposed to get the benefits of computation offloading from mobile nodes to external servers while limiting the use of distant servers whose higher latency could negatively impact the user experience. For this architecture, we consider a usage scenario where no central authority exists and multiple non-cooperative mobile users share the limited computing resources of a close-by cloudlet and can selfishly decide to send their computations to any of the three tiers. We define a model to capture the users interaction and to investigate the effects of computation offloading on the users' perceived performance. We formulate the problem as a generalized Nash equilibrium problem and show existence of an equilibrium. We present a distributed algorithm for the computation of an equilibrium which is tailored to the problem structure and is based on an in-depth analysis of the underlying equilibrium problem. Through numerical examples, we illustrate its behavior and the characteristics of the achieved equilibria.

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Optimization Tools for Solving Equilibrium Problems with Nonsmooth Data

Pappalardo M.

Abstract: The paper deals with the gap function approach for equilibrium problems with locally Lipschitz data. The gap function inherits the locally Lipschitz continuity of the data. Hence, the connections between its generalized directional derivatives, monotonicity conditions on the equilibrium bifunction and descent properties can be analysed. In turn, this analysis leads to devise two descent methods. Finally, the results of preliminary numerical tests are reported.

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Game Theory in Pattern Recognition and Machine Learning

Torsello A.

Abstract: Game theory arose in the early 40's as a reaction against the view that problems in economic theory can be formulated with the tools of optimization theory. Indeed, most real-world economic problems typically involve conflicting interactions among agents that cannot be captured by a single objective function, resulting in a shift of emphasis by game theorists from optimality towards equilibrium conditions. Game theory provides an abstract theoretically-founded framework to model complex scenarios, finding a variety of applications no only in economics and social sciences, but also in different fields of engineering and information technologies. Indeed, in the last 30 years there have been various attempts aimed at formulating problems in computer vision, pattern recognition and machine learning from a game-theoretic perspective. Further, with the recent development of algorithmic game theory, the interest in these communities around game-theoretic models is growing at a fast pace. In this talk I will provide an overview of the main applications of game theory in computer vision, pattern recognition and machine learning.

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CONTRIBUTED COMMUNICATIONS

On Simplified Hyperbolic Variational Inequality

Abdallah B.

Abstract: Time dependent Signorini problems are usually modeled by hyperbolic or parabolic variational inequalities. The purpose of this work is to extend the study of J. U. Kim on a boundary thin obstacle problem for a wave equation to a simplified time dependent Signorini problem with a given friction. To prove the existence of a solution, we follow a basic procedure which consists of a penalty method, a multiplier technique and compensated compactness.

To this end. First, we define an approximate problem $(P(k))$ of the variational inequality (P) by penalization with penalty parameter k. Next, by regularization method, we define a smooth nonlinear equation $(P(k, e))$ approximating our problem with a small parameter. Next, we prove that this problem has a solution. After that, we establish a prior estimates of the solution with respect to this parameter. Then, we establish a prior estimates of the solution with respect to k. Finally, passing to the limit we get a weak solution of the initial problem (P).

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Performing a Sensitivity Analysis on Italian Innovative SMEs' Creditworthiness

Angilella S.

Abstract: This study presents an optimization procedure to evaluate firms' creditworthiness, using a sample of Italian innovative SMEs based on AIDA data set. The construction of the model is based on a Multiple Criteria Decision Aid (MCDA) approach, ELECTRE-TRI. The analysis stands on the simulation of different set of parameters characterizing the linear configuration of ELECTRE-TRI. First, on the basis of each decision scenario each firm is assigned to a risk class. Then we perform an optimization procedure to test if the risk class of each firm could be worsened by slightly changing the simulated set of parameters. The obtained results are analyzed to assess SMEs' credit risk and to give insights in the stability of the assignment of each firm to its initial risk class.

Based on a joint work with S. Mazzù.

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A Stochastic Formulation for Oligopolistic Market Equilibrium Problem with Excesses

Barbagallo A.

Abstract: The aim of the talk is to study, in a Hilbert space setting, a general random oligopolistic market equilibrium problem in presence of both production and demand excesses and characterize the random Cournot-Nash equilibrium distribution by means of a stochastic variational inequality. Some existence results are provided and the associated Lagrange function is studied. Finally, a numerical example explains the theoretical results.

Based on a joint work with G. Di Meglio and P. Mauro.

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Equilibria for Semi-infinite Programming

Bigi G.

Abstract: Bilevel optimization, noncooperative games and semi-infinite programming share some similarities, which may lead to meaningful connections. Indeed, theoretical developments and algorithms developed for one of these models could be exploited to cope with the others. In this talk we focus on the relationships between generalized Nash games and semi-infinite programming. In particular, we show how generalized Nash games can be exploited to solve semi-infinite programs with convex-concave constraints, relying on penalization techniques and a sequence of suitable saddlepoint problems.

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Variational Inequality Approach for the Study of an Exchange Economy with Incomplete Markets and Nominal Assets

Donato M.B.

Abstract: We deal with the analysis of the general equilibrium model with incomplete financial markets and nominal assets. We assume that there are 2 periods of time, say today and tomorrow. The time structure of the model is the following one. In period zero, households exchange goods and assets and then consumption takes place. In period 1, one of S possible states of nature occurs. In each of them, assets pay their returns, households exchange goods and then consumption takes place. For given endowments, prices and returns, households maximize their utilities under a period by period budget constraint. We assume that financial markets are incomplete, that is the rank of the return matrix is smaller than the number of states in period 1. We can then define a consumption, portfolio holding, commodity and asset price vector as an equilibrium vector associated with a given economy if at those prices and economies households maximize, and market clears, i.e., commodities and assets supply and demand are equal. We provide a variational formulation of this economic problem and we give an existence proof of the equilibrium in terms of a quasi-variational inequality.

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Multiplicity Results for a Semilinear Elliptic Problem with a Singular Term

Faraci F.

Abstract: Singular problems arise in the study of non-Newtonian fluids, boundary layer phenomena for viscous fluids, chemical heterogeneous catalysts, as well as in the theory of heat conduction in electrically conducting materials. An increasing attention to singular stationary or evolution equations has been paid in the last decades.

In the present talk we deal with a singular elliptic problem of the following type:

$$(P) \qquad \begin{cases} -\Delta u = f(u) + u^{-\gamma} & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{su } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $0 < \gamma < 1$, $f : [0, +\infty[\to \mathbb{R}$ is a continuous function.

According to the value of the limit

$$\lim_{t\to\infty} \frac{f(t)}{t},$$

we prove different multiplicity results. We point out that the energy functional associated to (P) is not Gâteaux differentiable in the Sobolev space $W_0^{1,2}(\Omega)$ and the classical critical point theory does not apply. In our proof truncation techniques, variational arguments, Morse theory and direct minimizaton methods are employed.

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Controllability of Nonlinear Parabolic Equations

Floridia G.

Abstract: In this talk, we study the global approximate controllability of nonlinear parabolic equations via bilinear controls. In particular, we discuss multiplicative controllability results for semilinear reaction-diffusion equations governed via the coefficient of the reaction term. In the one dimensional case, we extend some nonnegative controllability results by bilinear controls assuming that both the initial and target states admit no more than finitely many changes of sign. These results permit to approach the m-dimensional case with radial symmetry. Indeed, assuming that the initial and final data are radial on a ball about the origin, we reduce the problem to a one-dimensional setting. In this way, we obtain an approximate controllability result for data which admit finitely many hyperspheres of sign change. Our interest in reaction-diffusion problems is partly motivated by the study of mathematical models for tumor growth.

Based on a joint work with P. Cannarsa and A. Khapalov.

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Distributed Consensus-type Algorithms to find an Epsilon-Nash Equilibrium of Aggregative Games with Coupling Constraints

Gentile B.

Abstract: We analyze deterministic aggregative games with large but finite number of players that are subject to both local and coupling constraints. Firstly, we derive sufficient conditions for the existence of a generalized Nash equilibrium, by using the theory of variational inequalities together with the specific structure of the objective functions and constraints. We present a centrally-coordinated scheme, belonging to the class of asymmetric projection algorithms, and we prove its convergence to a generalized Nash equilibrium by extending the results in the literature. Secondly, we consider a setup where the role of the central coordinator is replaced by communications via a network and thus introduce network aggregative games with coupling constraints; we show how an epsilon-generalized Nash equilibrium of the original aggregative game can be reconstructed through a distributed algorithm that leverages on the network aggregative game. Our theoretical results are applied to the problem of charging a fleet of plug-in electric vehicles.

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Solutions of Quasi-variational Inequalities Through Variational Inequalities

Milasi M.

Abstract: The aim of this talk is to identify a class of quasi-variational inequalities for which each solution of an auxiliary (classical) Stampacchia variational inequality provides a solution of the quasi-variational inequality. This class of quasi-variational inequalities is directly inspired by an equilibrium problem for economies involving sequential trade under conditions of uncertainty. A model with these features is represented by the Radner equilibrium. Existence of solutions for this class of quasi-variational inequalities is then deduced.

Based on a joint work with D. Aussel, M.B. Donato and A. Sultana.

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Decomposition Method for Oligopolistic Competitive Models with Common Environmental Regulation

Oggioni G.

Abstract: We consider the general problem of industrial production in a set of countries subject to a common environmental regulation that limits the emissions of specific sectors. Due to these restrictions, the problem is treated as a generalized non-cooperative game where players (countries) have joint (environmental) constraints caused by the necessity of a common and compulsory emission regulation. The problem is to find a natural mechanism for attaining the corresponding generalized equilibrium state. We suggest a share allocation method, which yields a suitable decomposition type procedure and replaces the initial problem with a sequence of non-cooperative games on Cartesian product sets. We also show that its implementation can be simplified essentially after application of a regularized penalty method. In the case study, we take inspiration from the European Union Emission Trading System (EU-ETS) and we introduce an environmental regulation that restricts the carbon emissions of energy, cement, and steel sectors in some European countries. Our results confirm the important role played by energy sector in reducing carbon emissions.

Based on a joint work with E. Allevi, A. Gnudi and I.V. Konnov.

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On the Convergence of Decentralized Algorithms to the Traffic User Equilibrium in a Road Network with Dynamics

Paccagnan D.

Abstract: We consider a network and a large number of agents, each of them willing to drive from an origin node to a destination node in the network, subject to dynamics given by the cell transmission model. Each agent is selfish and chooses his route and departure time to minimize his cost function given by its travel time plus a discomfort due to earliness or lateness of arrival. We are interested in decentralized algorithms that converge to a traffic user equilibrium. Similar models have been investigated in the traffic literature, where different algorithms converge in practice to the user equilibrium, but often no theoretical guarantees are provided. We formulate the traffic equilibrium problem as a variational inequality. In this preliminary work, we show monotonicity of the resulting operator, for the simplified setup of a short time horizon and a network composed by a single edge. As a consequence, many decentralized algorithms can be used to find a user equilibrium. Extensive simulations shows that convergence is achieved not only in the above simplified setup, but also for longer horizons and larger networks. Over the same one-edge network, we find the explicit expression of a user equilibrium for any horizon length, under more restrictive conditions on the cost functions.

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Service Provisioning Problem in Cloud and Multi-Cloud Systems

Passacantando M.

Abstract: Cloud computing is a new emerging paradigm that aims to streamline the on-demand provisioning of resources as services, providing end users with flexible and scalable services accessible through the Internet on a pay-per-use basis. This paper aims to study the hourly basis service provisioning problem through a generalized Nash game model. We take the perspective of Software as a Service (SaaS) providers that want to minimize the costs associated with the virtual machine instances allocated in a multiple Infrastructures as a Service (IaaS) scenario while avoiding incurring penalties for execution failures and providing quality of service guarantees. SaaS providers compete and bid for the use of infrastructural resources, whereas the IaaSs want to maximize their revenues obtained providing virtualized resources. We propose a solution algorithm based on the best-reply dynamics, which is suitable for a distributed implementation. We demonstrate the effectiveness of our approach by performing numerical tests, considering multiple workloads and system configurations. Results show that our algorithm is scalable and provides significant cost savings with respect to alternative methods (5% on average but up to 260% for individual SaaS providers). Furthermore, varying the number of IaaS providers means an 8%-15% cost savings can be achieved from the workload distribution on multiple IaaSs.

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Dirichlet Problems with Singular Convection Terms

Zecca G.

Abstract: We study the unilateral obstacle problem related to second order nonlinear elliptic equations whose model appears in the stationary diffusion-convection problem. We obtain existence, uniqueness and regularity of solutions when the growth coefficient of the convection term lies in the Marcinkiewicz space *weak*-$L^N(\Omega)$.

Based on a joint work with L. Greco and G. Moscariello.

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