STATISTICAL INFERENCE FOR IMAGE RECONSTRUCTION THROUGH MULTIMODE FIBERS

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Abstract.

A major interest in biomedical imaging is the comprehension of the photon scattering through disordered media[1]: Many studies have pursued the solution of this riddle achieving light-focusing control[2] or for reconstructing images[3] in complex media. In the present work, we investigate how statistical inference could help the calculation of the transmission matrix in a multimode fiber, thus enabling its usage as a normal optical element. Our desire is to uncover insights from the scattering problem, encouraging the development of imaging techniques for better medical investigations.

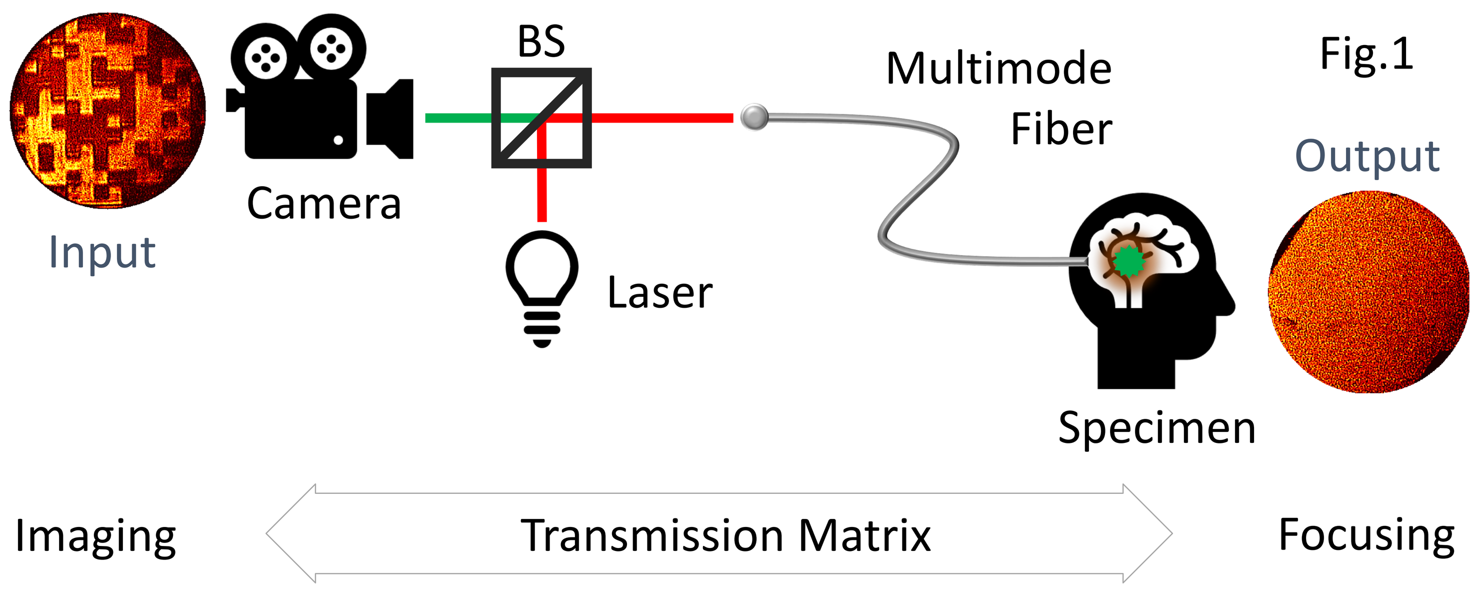

Fig.1

Theoretical Background.

The experimental setup that we examine is illustrated in Fig.1. Light scrambling due to the disordered structure of a multimode fiber impedes direct imaging and light-control by manipulating the field in one of the fiber ends. It is fundamental to estimate how the fiber acts on the input field to compensate for the loss of information due to the randomicity of the scattering event occurring in the process. We assume that input and output are coupled by a transmission matrix T of the form:

$$I_i^{out} = \sum_{j=1}^{N} T_{ij} I_j^{in} \longrightarrow \mathcal{H} = \beta \sum_{i,j=1}^{2N} I_i J_{ij} I_j$$

$$J = \begin{bmatrix} -\mathbb{I} & T' \\ T & -TT' \end{bmatrix} \quad \text{with} \quad I = \begin{bmatrix} I^{in} \\ I^{out} \end{bmatrix}$$

Allowing statistical noise fluctuations, it is possible to express the Hamiltonian of the system in a form similar to the Ising or XY case[4], where the global coupling matrix J depends striclty on T.

Statistical Methods.

We want to infer the couplings that most likely represents the acquired input and output data. Due to the problem complexity, it is convenient to approximate the calculation of the likelihood of the system with the pseudolikelihood[5,6] (PSL) of each intensity I_i given all the others $I_{\setminus i}$. We rewrite the Hamiltonian so that the probability that a realization of I_i given $I_{\setminus i}$ it is expressed as:

$$\mathcal{H} = \sum_i I_i \sum_j J_{ij} I_j = \sum_i \hbar_i \qquad \mathcal{P}(I_i)_{\setminus i} = \frac{e^{\hbar_i}}{\mathcal{Z}_i} \quad \text{with } \mathcal{Z}_i = \int_0^{\infty} dI_i \, e^{\hbar_i}$$

In this formulation, the matrix that likely describes the couplings of the variables I is the one that maximizes each probability $\mathcal{P}(I_i)_{\setminus i}$, so that:

$$\mathcal{L}_i = \ln \mathcal{P}(I_i)_{\setminus i} \qquad \mathcal{L} = \frac{1}{N_\mu N} \sum_{\{\mu\}} \sum_i \mathcal{L}_i$$

Explicitly, each $\mathcal{L}_i$ can be calculated analytically with the following expression:

$$\mathcal{L}_i = I_i \sum_j J_{ij} I_j + \mu I_i^2 + \frac{1}{2} \ln \frac{\pi}{4J_{ii}} + \ln \left[\mathrm{erfc}\left(\frac{\sum_j J_{ij} I_j}{\sqrt{4J_{ii}}} \right) e^{\frac{\left(\sum_j J_{ij} I_j\right)^2}{4J_{ii}}} \right]$$

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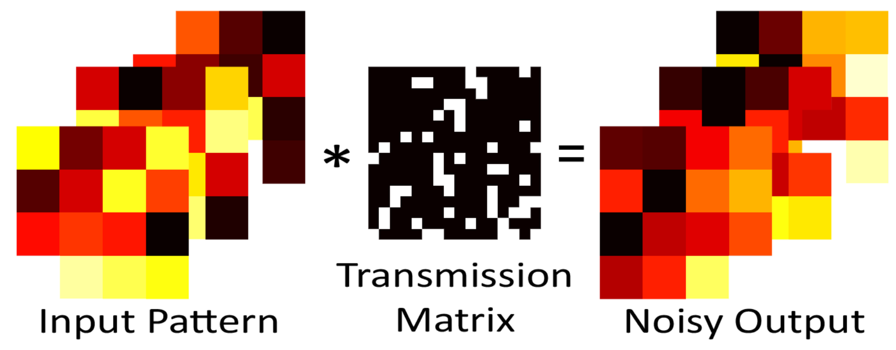
Results.

Assuming a linear input-output process, we test numerically the PSL maximization procedure. We create a random transmission matrix acting on $N_\mu = 1000$ randomly generated patterns composed by $N = 16$ pixels.

Fig.2



The corresponding outputs were perturbed with 2% gaussian noise, to replicate experimental conditions. Maximizing $\mathcal{L}$ with free-parameters returns a generic coupling matrix J having in the expected form (Fig.3).

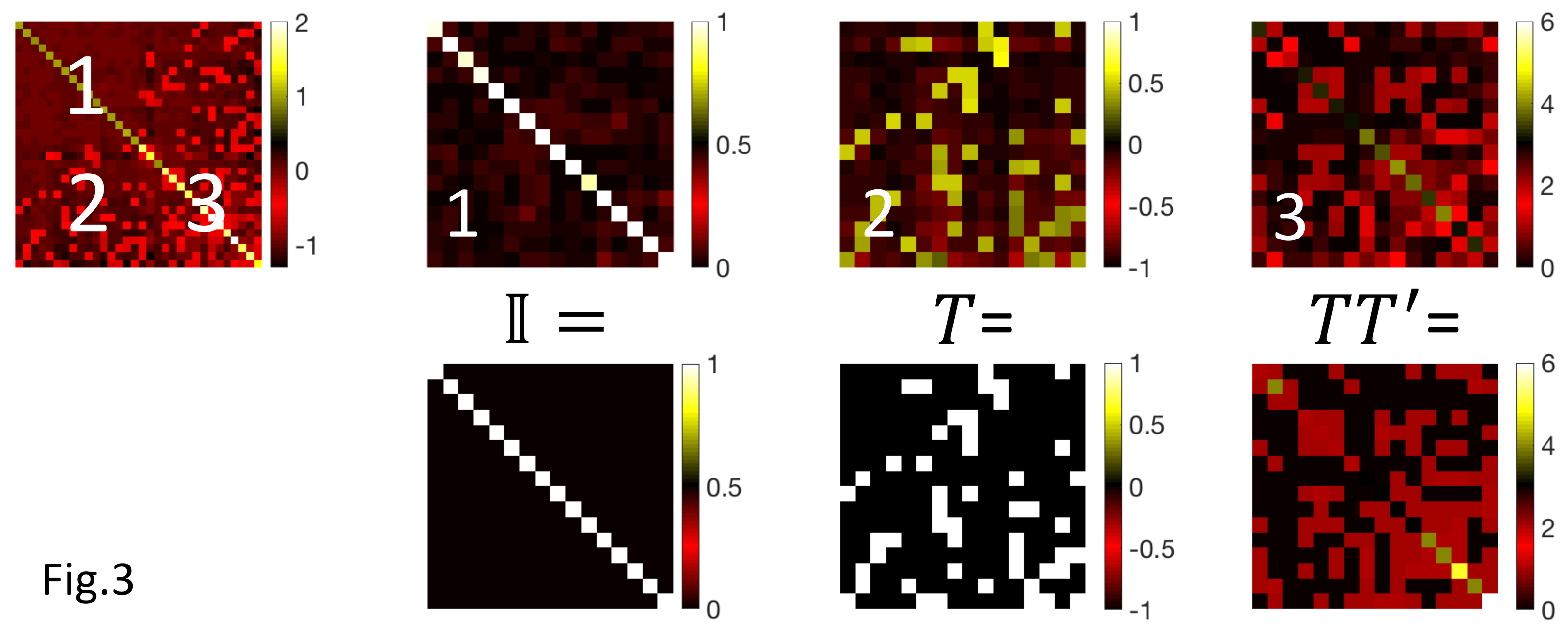
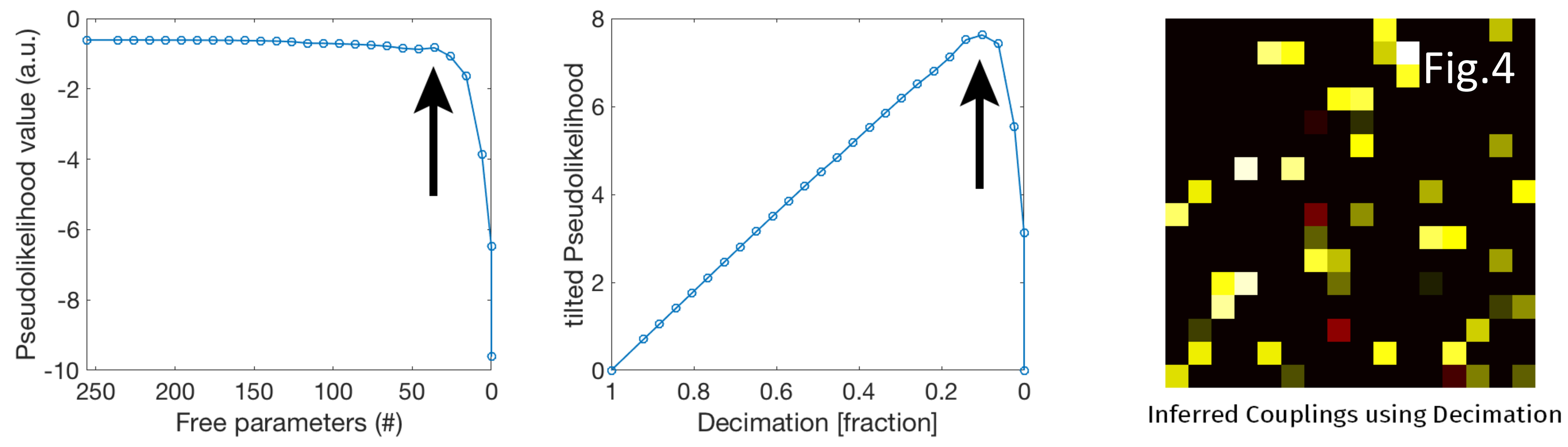


Fig.3

Perspectives.

It is possible to infer more accurately the coupling elements effectively present in the transmission matrix. Decimation procedure[7] can better estimate the number of parameters by maximizing the tilted $t\mathcal{L} = \mathcal{L} - x\mathcal{L}_{max} - (1-x)\mathcal{L}_{min}$ where x is the fraction of decimated couplings.



The Hamiltonian reformulation of the transmission problem allows the inference of the transmission matrix via the maximization of the PSL function. The problem is still complex, in particular for a high number of free parameters, but decimation procedure potentially offer opportunity to achieve high accuracy and simplify the problem to fewer, yet more relevant, couplings. We are going to test the procedure with experimental data, refining the calculation of the PSL to take into account also the phase of the fields. Phase in fact cannot be directly sensed by camera, but can be taken into account in our formulation, leaving room for further advancing of the technique.

References.

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